CREATION OF DEM BY KRIGING METHOD AND EVALUATION OF THE RESULTS

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The generated DEM is used for deriving values of various morphometric variables of relief; however, their values differ according to the parameters of interpolation methods used. An improperly selected and set interpolation method results in the creation of a DEM of low quality, which then results in the derivation of erroneous values of geomorphometric parameters. Errors in derived parameters are usually much more evident than in the original DEMs. This is further enhanced by the properties (configuration) of the real relief – plains, hilly lands, highlands and mountains ("relief" in a following text means the relief of the Earth's surface). It is evident that there is direct proportion between the relative relief segmentation and the examined inaccuracies – in hilly areas the inaccuracies are smaller than in mountainous areas. Any error in the DEM then generates an error in the application results where relief is one of the factors.

Particularly the process of testing the kriging method as an interpolation method for the creation of DEMs of various types of relief - plains, hilly lands, highlands and mountains will be present under the terms of this article. Input data, which were used for the testing, were the layers with the altitudinal data of DMU25, which were obtained from the Department of geoinformatics at the Palacký University in Olomouc and the research grant MŠMT with the name "Dynamical Geovisualization in Critical Management" solved at the Institute of geography at the Masaryk University in Brno.

Keywords: interpolation, kriging, DEM, relief

BASIS OF KRIGING THEORY

The kriging is based on the theory of the regionalized variable which assumes that the special variability of a phenomenon expressed by *Z* values is statistically homogenous within the whole territory (DUTTER 2000). The kriging involves a set of methods; the ordinary and universal krigings are widely recognized as the ones of the fundamental kriging types.

The ordinary kriging exploits various methods, for example spherical, circular, exponential, Gauss and/or linear method. These methods and/or mathematical functions are used for fitting of data depicted by a line or curve in the experimental semi-variogram. The ordinary kriging also assumes that the variability of *Z* values is independent on any structural component (i.e. drift). On the other hand, the universal kriging assumes that the spatial variability of *Z* values is composed of three parts (HOULDING 2000, DIGGLE and RIBEIRO 2007):

- a drift (i.e. a structural component which expresses a constant trend of a surface);
- a random but spatially correlatable variable;
- random noises (spatially independent with an assumption of normal distribution).

It is suitable to use the universal kriging in cases when the existence of the local trend in data is assumed that can manifest itself as a moderate parabolic (progressive) curvature of the semi-variogram shape around the proximity of the origin. In the opposite case, the ordinary kriging is advised to be rather exploited.

The character of the spatially correlated variability can be demonstrated by the semi-variogram (semi-variance is considered as a degree of variance), which provides information on optimization of interpolation weights and seek radiuses. DUTTER (2000) introduces a relation for calculation of semi-variance by means of an arithmetical average of the squares of the differences between the two experimental measurements, denoted by $[z(x_i), z(x_i + h)]$, performed in the corresponding two points separated by the vector (distance) h, i.e.

$$\gamma(h) = \frac{1}{2n}\sum_i \left[z(x_i) - z(x_i + h)\right]^2 ,$$

where $\gamma(h)$ stands for the semi-variance of Z variable for distance h (h corresponds to the magnitude of lag) and $n(h)$ denotes the number

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of pairs of points, [$z(x_i)$, $z(x_i + h)$], with a distance h.

INTERPOLATION WEIGHTS

The as obtained values of semi-variance are put into the semi-variogram (**Fig. 1**) from which required values of interpolation weights can be derived – e.g. sill, range, nugget (ARMSTRONG 1998). The sill parameter (i.e. threshold) corresponds to the overall dispersion (i.e. semi-variance). If flat part exists behind this threshold level in the semi-variogram, it means that on further increase of the distance, the values of the variance do not change (i.e. they are constant). The distance for which the semi-variance reaches the threshold value (i.e. the range) expresses the degree of correlation within the dataset. The long distance indicates a high correlation whereas the short distance implies for a low correlation. The experimental semi-variograms often cross the y-axis at some non-zero value which is called as a nugget or nugget effect (i.e. the residual dispersion).

POSSIBILITIES OF INTERPOLATION IN ARCGIS

Digital moddels of the relief by interpolation kriging method can be created in the environment of ArcGIS v. 9.3 by the tools of 3D Analyst extension (similar with Spatial Analyst extension) and/or by Geostatistical Analyst extension. The interpolation tool using the first localization (3D Analyst extension → Interpolate to raster… → Kriging) offers a possibility of interpolation by means of the ordinary kriging with a use of five different methods (models), namely by spherical, circular, Gaussian, exponential and linear method, or by means of the universal kriging with or without a linear trend. In addition we can use a dialog window for setting of Advanced parameters which represent the above-mentioned interpolation weights (i.e. range, sill and nugget). The disadvantage is that when using this way of setting the parameters, the semi-variogram is not continuously displayed so that it is not possible to monitor the suitability of settings of the selected method for fitting the data with the model curve. The choice of known points entering into the interpolation procedure can be realized with the exploitation of the seek radius and/or the number of the nearest points (without any opportunity of preferring the certain direction).

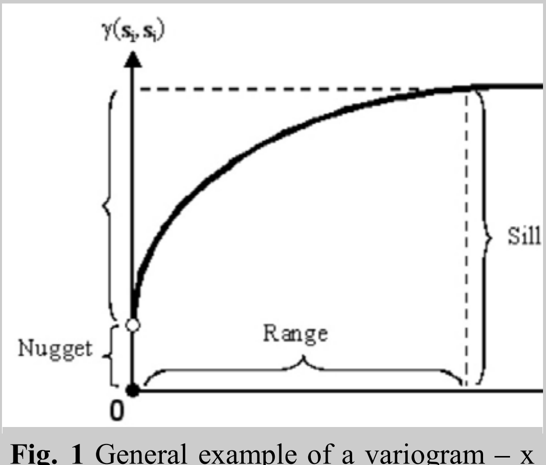

Fig. 1 General example of a variogram – x axis = distance (lag), y axis = semivariation (source: ARCGIS 9.2 DESKTOP HELP 2009)

When using the tool for interpolation within the scope of the Geostatistical Analyst extension (Geostatistical Analyst extension → Geostatistical Wizard… → method: kriging), other methods can be used except the ordinary and universal kriging. Within the ordinary kriging, which have been exploited for the creation of digital models of the relief, we can use not only five models offered by the 3D Analyst extension but eleven different models in total. In addition, both the semi-variogram constructed from the input data and the curve of the selected model are displayed (**Fig. 2a**). When setting the interpolation weights, it is possible to directly monitor the changes in the profile of this curve and thus to achieve the best fitting of the experimental semi-variogram. Other advantage of this extension also lies in the possibility of choice between selection of known points entering into interpolation originating from any direction or from selected quadrants and octants (**Fig. 2b**).

EVALUATION OF ACCURACY

THE EXISTING METHODS OF EVALUATION OF ACCURACY

The root mean square error (RMSE) is the most frequently used characteristics determining the degree of accuracy. It expresses the dispersion of distribution of frequency of variances between original height data and DEM data. In a mathematical speaking, it is expressed as

$$\mathrm{RMSE}_Z = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Z_{di} - Z_{ri}\right)^2},$$

where Z_{di} is the i-th value of the altitude above DEM surface, Z_{ri} is the corresponding original altitude and n is the number of the controlled points.

The higher value of RSME corresponds to the larger dispersion between the two datasets.

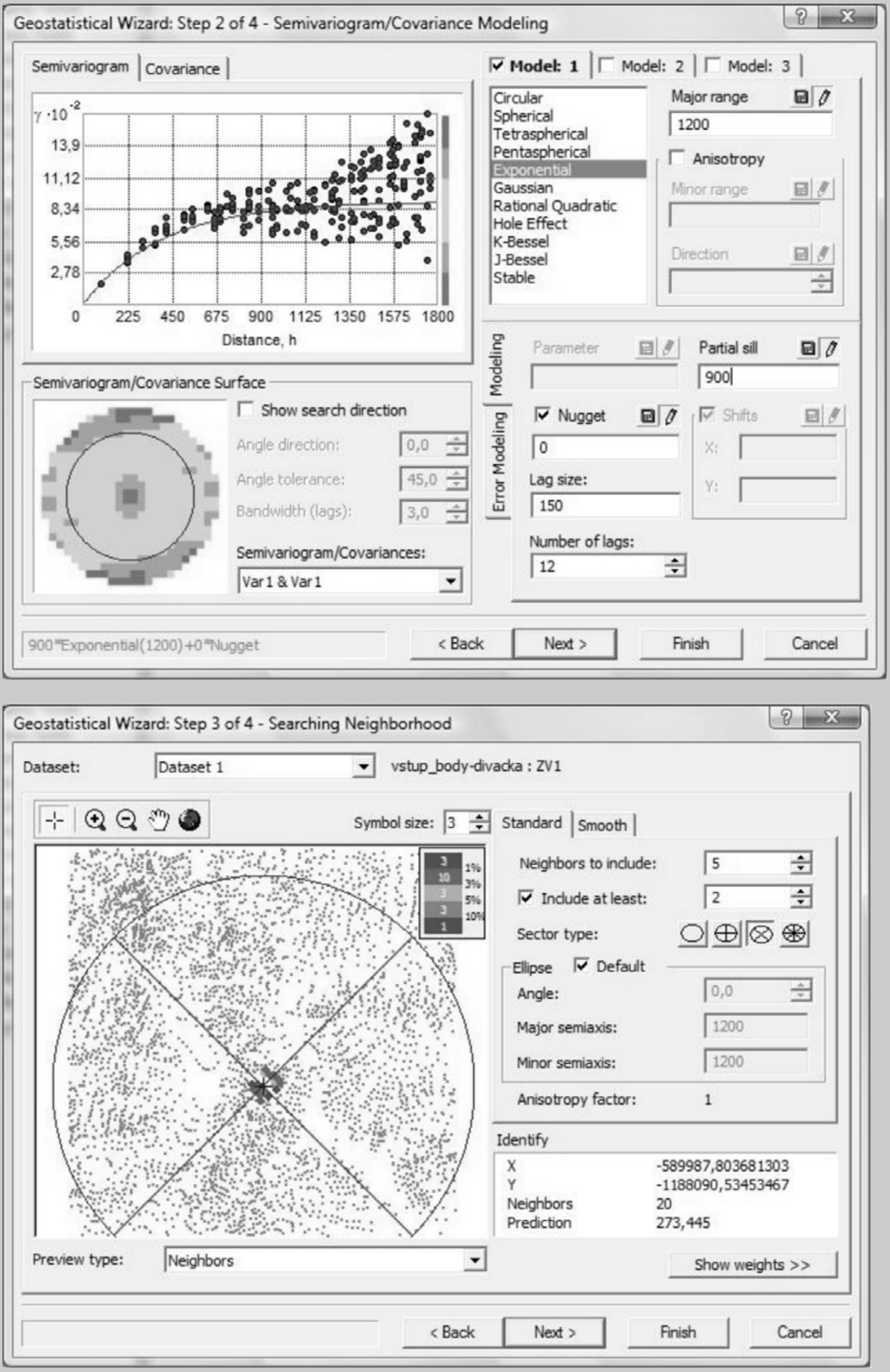

Fig. 2 Options for the interpolation parameters setting in the environment of the Geostatistical Analyst extension: a) setting of parameters range, sill and nugget, b) setting of number of sections and neighbours (source: Geostatistical Analyst extension)

The ideal value should not overcome the half of the value of the interval of the original contour lines. Its global expression within the whole territory is considered to be the only drawback of this characteristics.

On the contrary, the hammock index (*H*) is the less commonly used characteristics for a determination of the quality of interpolation. It monitors the uniformity of distribution of interpolated values between known values and excessive occurrence of pixels with an altitude corresponding to the value of original data. The calculation of this characteristics is performed especially in cases when the counter lines are the input data. Obtaining of the value of the hammock index is based on the conversion of DEM to the integer grid and on calculation of the module (i.e. residue after integer division) when the divider agrees with a number corresponding to the interval of original contour lines. The distribution of module values can be visualized in the form of a surface (i.e. grid) and/or histogram and is called the hammock plot. The global value (or the eigenvalue) of the hammock index is calculated by (WOOD 1996)

$$H = \frac{(nf_0) - \sum_{i=1}^{n-1} f_i}{\sum_{i=0}^{n-1} f_i},$$

where n stands for the interval of the original contour lines, f_0 is the mod0 frequency (i.e. the frequency of pixels with 0 residue) and f_i represents the frequency of other modes.

The hammock index ranges in the interval of <–1, *i* – 1>. The ideal values that express well-balanced modulo, i.e. well-balanced frequency in all considered intervals and uniform distribution of interpolated values between input data, are found to occur around zero.

Extension of the used methods

Commonly referred limit value of RMSE is 1/2 of the input interval values. In the case of the dataset from DMU25 is the map line interval 5m and the limit value of RMSE is 2,5m. According to the fact that different types of relief with different height levels are evaluated, it is very important to clarify the importance of the error (2.5m). (**Tab. 1**).

The value of RMSE 2.5m defines a very precise DEM (close to the original one) in the case of mountains and highlands. Especially in the case of mountains the transformation of the RMSE to the percentage form gives us the result that the error is about 0,417 — 0,556 % (100 % belongs to the relative height levels of the given area). Another situation could be observed in the case of plains and wolds. The limit value of the RMSE represents the more important error, which is higher than 8,334 %. In other words, the more ragged is the relief the smaller is the importance of the limit value of the RMSE. The limit value of the RMSE seems to be less important for mountains like for plains.

It is possible to convert the limit value of RMSE to the percentage form in order to make the evaluation of the DEM more objective. This value could be than converted to the value in meters individually for all types of relief. This procedure has one disadvantage. We would probably not be able to obtain such a good results in the case when the percentage accuracy comes from the limit values defined for the mountains (for example 0.417 %, see **Tab. 1**) which is than recomputed to the limit

Type	Relative segmentation [m]	RMSE [m]	RMSE [%]	RMSE [%]	RMSE [m]	RMSE [%]	RMSE [m]
plain	0,001	2,500	250000,000	1,667	0,000	0,417	0,000
	30	2,500	8,333	1,667	0,500	0,417	0,125
fl. wold	30	2,500	8,333	1,667	0,500	0,417	0,125
	75	2,500	3,333	1,667	1,250	0,417	0,313
rug. wold	75	2,500	3,333	1,667	1,250	0,417	0,313
	150	2,500	1,667	1,667	2,500	0,417	0,625
fl. highlands	150	2,500	1,667	1,667	2,500	0,417	0,625
	225	2,500	1,111	1,667	3,750	0,417	0,938
rug. highlands	225	2,500	1,111	1,667	3,750	0,417	0,938
	300	2,500	0,833	1,667	5,000	0,417	1,250
fl. mountains	300	2,500	0,833	1,667	5,000	0,417	1,250
	450	2,500	0,556	1,667	7,500	0,417	1,875
rug. mountains	450	2,500	0,556	1,667	7,500	0,417	1,875
	600	2,500	0,417	1,667	10,000	0,417	2,500

Tab. 1 Setting of limitary values of RMSE for the different types of relief

value of RMSE in meters for different types of relief. It is more appropriate to set the lower level of accuracy (for example 1.667 %, see **Tab. 1**).

The next characteristics, which has been used for the post evaluation of DEM is the absolute error – AE. This value is the real sum of deviation from the reference grid in absolute values. The formula is given by

$$AE = \sum_{i=1}^{n} \left| Z_{di} - Z_{ri} \right|,$$

where: Z_{di} is the i value of the altitude obtained from the DEM

Z_{ri} is the corresponding altitude (original).

The computation of more characteristics for the evaluation allows us to make the comparison and aggregation. We are not able to confirm the rule that the smaller is the RMSE the smaller is the absolute error and the weighted hammock index. The comparison and aggregation of the obtained results should be therefore done with the use of the method called "The aggregation of the weighted order" designed and described by TALAŠOVÁ (2003). It is the method of the simple weighted order which is well known in every kind of decision processes, but the advantages of this method (proved in TALAŠOVÁ 2003) is in the more sophisticated and credible results of the decision process.

The first step in the obtaining of the weighted order is to set the normal weights (importance) of all characteristics. The weights were set in the process of DEM evaluation as follows: RMSE - 2/5, AE - 2/5 and Hammock index - 1/5. The second step is the computation of the DEM order obtained with the help of the above given weights.

DEM CREATED WITH THE HELP OF THE KRIGING METHODS

The aim of this article is not only describe and lead the reader through the principles of the kriging as a special interpolation tool for DEM creation. The deeper task is to evaluate the setting of this method compared with the resulted accuracy of the obtained models. This all is based on the above given characteristics like RMSE, AE and Hammock index.

The dataset of DMU25 was used for the DEM creation. These data were obtained from the Department of geoinformatics at the Palacký University in Olomouc and the research grant MŠMT with the name "Dynamical Geovisualization in Critical Management" solved at the Institute of geography at the Masaryk University in Brno. Digital elevation models were created for four selected areas with different types of relief. Each has the area of 4 km^2. The setting of the parameters for each area is given in **Table 2**. Ordinary kriging with the four different types of variograms were used for the creation of DEMs (circular, spherical, exponential and Gaussian). The names of variograms have named also the resulted DEMs. The names with the index 1 correspond to the DEMs created with the default setting. The names with the index 2 correspond to the DEMs, where parameters were set according to the subjective estimation, which was made by the user according to the shape of the semi-variogram curve. The number of lags was set to the value of 12 in all cases, the number of nearest known neighbors was also set to the value of 12 and the number of sector used for the selection of the neighbors was set to the number of 4.

EVALUATION OF FINAL DEMS

For all 32 digital models of the relief, we determined the values of the above-mentioned statistical characteristics (RMSE, absolute error and hammock index) that assess the quality of the interpolation results (**Tab. 3**). For each territory (i.e. a type of the relief), the created DEMs were arranged according to the weighted order.

The commonly indicated limit value of RMSE = 2.5 m has not been overcome except 3 cases from 32 DEMs (for various types of the relief). The exceeding of order of 1-2 tenths was observed for cases when using the Gaussian model for hilly country (kr_gau_1) and for highlands (kr_gau_1, kr_gau_2). If more strict criteria (0.417 %), derived from the percentage expression of RMSE (**Tab. 1**), are taken into consideration then the limit values exceed even the both DEMs created with the use of Gaussian model for lowlands and all DEMs created by any model for hilly countries and highlands. On the contrary, all digital models of the relief for uplands fulfill these more strict criteria.

The magnitude of the absolute error is equable for most of DEMs for particular types of the relief. Only DEMs created with the use of the Gaussian model again exhibit a bigger absolute error, especially for highlands and uplands. In summary, one can say that the order based on the absolute error well corresponds to the order deduced from the values of RMSE for individual DEMs.

The values of the hammock index close to zero indicate the balance of individual modulo

Sadská plain					
Name	**Model**	**Range**	**Sill**	**Nugget**	**Lag size**
kr_cir_1	circular	1,527,825	4,389	0,000	182,420
kr_cir_2	circular	800,000	3,000	0,000	75,000
kr_sph_1	spherical	1,709,485	4,320	0,000	182,420
kr_sph_2	spherical	850,000	2,500	0,000	75,000
kr_exp_1	exponential	2,161,273	4,228	0,000	182,420
kr_exp_2	exponential	700,000	2,000	0,000	75,000
kr_gau_1	gaussian	1,184,664	3,537	0,407	182,420
kr_gau_2	gaussian	900,000	1,800	0,200	75,000
Podještědská wold					
Name	**Model**	**Range**	**Sill**	**Nugget**	**Lag size**
kr_cir_1	circular	2,019,910	431,730	68,074	183,340
kr_cir_2	circular	500,000	180,000	40,000	75,000
kr_sph_1	spherical	2,173,178	423,010	62,294	183,340
kr_sph_2	spherical	500,000	180,000	30,000	75,000
kr_exp_1	exponential	2,173,178	443,887	13,621	183,340
kr_exp_2	exponential	750,000	230,000	20,000	75,000
kr_gau_1	gaussian	1,938,456	386,874	120,227	183,340
kr_gau_2	gaussian	500,000	150,000	70,000	100,000
Bozkovská highlands					
Name	**Model**	**Range**	**Sill**	**Nugget**	**Lag size**
kr_cir_1	circular	2,173,178	5,875,105	49,109	183,340
kr_cir_2	circular	800,000	2,700,000	0,000	100,000
kr_sph_1	spherical	2,173,178	5,388,979	0,000	183,340
kr_sph_2	spherical	900,000	2,500,000	0,000	75,000
kr_exp_1	exponential	2,173,178	4,711,949	0,000	183,340
kr_exp_2	exponential	1500,000	3,200,000	0,000	100,000
kr_gau_1	gaussian	2,040,094	5,346,704	676,790	183,340
kr_gau_2	gaussian	800,000	2,500,000	250,000	75,000
Hornoopavská mountains					
Name	**Model**	**Range**	**Sill**	**Nugget**	**Lag size**
kr_cir_1	circular	2,170,452	17,008,761	0,000	183,110
kr_cir_2	circular	1,300,000	9,000,000	0,000	120,000
kr_sph_1	spherical	2,170,452	15,493,271	0,000	183,110
kr_sph_2	spherical	1,200,000	8,000,000	0,000	100,000
kr_exp_1	exponential	2,170,452	14,259,827	0,000	183,110
kr_exp_2	exponential	1,100,000	7,000,000	0,000	100,000
kr_gau_1	gaussian	2,170,452	20,308,203	447,998	183,110
kr_gau_2	gaussian	1,100,000	9,000,000	500,000	150,000

Tab 2 Setting of parameters of the kriging method for the different types of relief

Sadská plain							
Name	**RMSE [m]**	**Order**	**Absolute error [m]**	**Order**	**H**	**Order**	**Weighted order**
kr_cir_1	0,079	1	10,709,532	1	4,586	2	**1,2**
kr_sph_1	0,079	1	10,728,186	2	4,586	2	**1,6**
kr_cir_2	0,079	1	10,809,083	3	4,586	2	**2,0**
kr_sph_2	0,080	2	10,883,911	4	4,586	2	**2,8**
kr_exp_1	0,082	3	10,984,179	5	4,594	3	**3,8**
kr_gau_2	0,150	5	11,113,596	6	4,584	1	**4,6**
kr_exp_2	0,087	4	11,541,985	7	4,614	5	**5,4**
kr_gau_1	0,168	6	11,886,988	8	4,606	4	**6,4**
Podještědská wold							
Name	**RMSE [m]**	**Order**	**Absolute error [m]**	**Order**	**H**	**Order**	**Weighted order**
kr_exp_1	1,522	1	74,179,312	1	0,484	8	2,4
kr_exp_2	1,532	2	75,281,016	2	0,476	7	3,0
kr_sph_2	1,798	3	110,071,346	3	0,300	6	3,6
kr_cir_2	1,948	4	13,2513,691	4	0,205	5	4,2
kr_sph_1	2,203	5	173,946,655	5	0,144	2	4,4
kr_cir_1	2,240	6	180,184,287	6	0,143	1	5,0
kr_gau_2	2,468	7	213,370,756	7	0,151	3	6,2
kr_gau_1	2,563	8	236,068,008	8	0,198	4	7,2
Bozkovská highlands							
Name	**RMSE [m]**	**Order**	**Absolute error [m]**	**Order**	**H**	**Order**	**Weighted order**
kr_sph_1	1,335	2	74,598,346	1	0,244	4	**2,0**
kr_sph_2	1,334	1	74,687,853	3	0,243	3	**2,2**
kr_cir_2	1,335	2	74,648,133	2	0,243	3	**2,2**
kr_exp_1	1,338	3	74,724,660	4	0,245	5	**3,8**
kr_exp_2	1,340	4	74,772,161	5	0,245	5	**4,6**
kr_gau_2	2,530	6	208,212,331	7	0,181	1	**5,4**
kr_cir_1	1,603	5	82,791,894	6	0,264	6	**5,6**
kr_gau_1	2,672	7	244,040,532	8	0,203	2	**6,4**
Hornoopavská mountains							
Name	**RMSE [m]**	**Order**	**Absolute error [m]**	**Order**	**H**	**Order**	**Weighted order**
kr_cir_1	1,212	1	79,132,187	1	0,199	1	**1,0**
kr_sph_1	1,212	1	79,138,761	2	0,199	1	**1,4**
kr_cir_2	1,212	1	79,143,289	3	0,199	1	**1,8**
kr_sph_2	1,212	1	79,172,033	4	0,199	1	**2,2**
kr_exp_1	1,215	2	79,424,911	5	0,199	1	**3,0**
kr_exp_2	1,219	3	79,679,753	6	0,199	1	**3,8**
kr_gau_2	2,003	4	147,479,293	7	0,219	2	**4,8**
kr_gau_1	2,049	5	159,283,780	8	0,224	3	**5,8**

Tab. 3 Statistical characteristics and the weighted order for the individual DEM

and thus uniform distribution of newly interpolated values between input known data. Besides digital models of the relief for lowlands, all acquired values of the hammock index (for DEMs of hilly countries, highlands and uplands) are in the range of 0.1 - 0.5. This confirms a quality interpolation to take place without excessive amount of pixels with the same altitude as the input contour lines had. In general, the best values have been achieved for DEMs created for uplands, which is probably related to the high density of input contour lines. Thus, it comes out why the values of the hammock index are conversely extremely high for lowlands as they are characteristic of very low density of input data.

CONCLUSION REMARKS

The evaluation of the resulted DEMs according to the different types of statistical characteristics and also according to the weighted order of the resulted DEMs gives us the conclusion that most of the DEMs created by the interpolation method ordinary kriging are very accurate and they meet also the requirements for the next computation (the computation of morphometric variables). Significantly different should be only DEMs created with the help of Gaussian model. The differences could be seen in the lower quality. This model of semivariogram could be also marked as a inappropriate for the creating of the DEMs. This statement could be valid for all experimental reliefs.

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